Q.11 – Q.35 Carry ONE mark Each

Q.11	Let $\boldsymbol{M}$ be a 2×2 real matrix such that $(\boldsymbol{I}+\boldsymbol{M})^{-1} = \boldsymbol{I} - \alpha\boldsymbol{M}$, where α is a non-zero real number and $\boldsymbol{I}$ is the 2×2 identity matrix. If the trace of the matrix $\boldsymbol{M}$ is 3, then the value of α is
(A)	$\frac{3}{4}$
(B)	$\frac{1}{3}$
(C)	$\frac{1}{2}$
(D)	$\frac{1}{4}$

Q.12	Let $\{X(t)\}_{t\geq 0}$ be a linear pure death process with death rate $\mu_i = 5i,\ \ i = 0, 1, \ldots, N,\ \ N \geq 1$. Suppose that $p_i(t) = P(X(t) = i)$. Then the system of forward Kolmogorov's equations is
(A)	$\frac{dp_i(t)}{dt} = 5(i+1)p_{i+1}(t) + 5i\, p_i(t)$ and $\frac{dp_N(t)}{dt} = 5Np_N(t)$ for $i = 0, 1, 2, \ldots, N-1$ with initial conditions $p_i(0) = 0$ for $i \neq N$, and $p_N(0) = 1$
(B)	$\frac{dp_i(t)}{dt} = 5(i+1)p_{i+1}(t) - 5i\, p_i(t)$ and $\frac{dp_N(t)}{dt} = -5Np_N(t)$ for $i = 0, 1, 2, \ldots, N-1$ with initial conditions $p_i(0) = 0$ for $i \neq N$, and $p_N(0) = 1$
(C)	$\frac{dp_i(t)}{dt} = 5(i+1)p_{i+1}(t) + 5i\, p_i(t)$ and $\frac{dp_N(t)}{dt} = 5Np_N(t)$ for $i = 0, 1, 2, \ldots, N-1$ with initial conditions $p_i(0) = 1$ for $i \neq N$, and $p_N(0) = 0$
(D)	$\frac{dp_i(t)}{dt} = 5(i+1)p_{i+1}(t) - 5i\, p_i(t)$ and $\frac{dp_N(t)}{dt} = -5Np_N(t)$ for $i = 0, 1, 2, \ldots, N-1$ with initial conditions $p_i(0) = 1$ for $i \neq N$, and $p_N(0) = 0$

Q.13	Let S^2 be the variance of a random sample of size $n > 1$ from a normal population with an unknown mean μ and an unknown finite variance $\sigma^2 > 0$. Consider the following statements: (I) S^2 is an unbiased estimator of σ^2, and S is an unbiased estimator of σ. (II) $\left(\frac{n-1}{n}\right) S^2$ is a maximum likelihood estimator of σ^2, and $\sqrt{\frac{n-1}{n}}\, S$ is a maximum likelihood estimator of σ. Which of the above statements is/are true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)

Q.14	Let $f: \mathbb{R}^2 \to \mathbb{R}$ be a function defined by $$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$ Then which one of the following statements is true?
(A)	f is bounded and $\frac{\partial f}{\partial x}$ is unbounded on $\mathbb{R}^2$
(B)	f is unbounded and $\frac{\partial f}{\partial x}$ is bounded on $\mathbb{R}^2$
(C)	Both f and $\frac{\partial f}{\partial x}$ are unbounded on $\mathbb{R}^2$
(D)	Both f and $\frac{\partial f}{\partial x}$ are bounded on $\mathbb{R}^2$

Q.15	Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with cumulative distribution function $F(x)$. Let the empirical distribution function of the sample be $F_n(x)$. The classical Kolmogorov-Smirnov goodness of fit test statistic is given by $$T_n = \sqrt{n}\, D_n = \sqrt{n} \sup_{-\infty<x<\infty} \lvert F_n(x) - F(x) \rvert.$$ Consider the following statements: (I) The distribution of T_n is the same for all continuous underlying distribution functions $F(x)$. (II) D_n converges to 0 almost surely, as $n \to \infty$. Which of the above statements is/are true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)

Q.16	Consider the following transition matrices $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ of two Markov chains: $$\boldsymbol{P_1} = \begin{bmatrix} 1 & 0 & 0 \\ 1/3 & 1/2 & 1/6 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{P_2} = \begin{bmatrix} 1/6 & 1/3 & 1/2 \\ 1/4 & 0 & 3/4 \\ 0 & 1 & 0 \end{bmatrix}.$$ Then which one of the following statements is true?
(A)	Both $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$ have unique stationary distributions
(B)	$\boldsymbol{P_1}$ has a unique stationary distribution, but $\boldsymbol{P_2}$ has infinitely many stationary distributions
(C)	$\boldsymbol{P_1}$ has infinitely many stationary distributions, but $\boldsymbol{P_2}$ has a unique stationary distribution
(D)	Neither $\boldsymbol{P_1}$ nor $\boldsymbol{P_2}$ has unique stationary distribution

Q.17	Let $\boldsymbol{X_1}, \boldsymbol{X_2}, \dots, \boldsymbol{X_{20}}$ be a random sample of size 20 from $N_6(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, with $\det(\boldsymbol{\Sigma}) \neq 0$, and suppose both $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are unknown. Let $$\overline{\boldsymbol{X}} = \frac{1}{20}\sum_{i=1}^{20} \boldsymbol{X}_i \quad \text{and} \quad \boldsymbol{S} = \frac{1}{19}\sum_{i=1}^{20} (\boldsymbol{X}_i - \overline{\boldsymbol{X}})\,(\boldsymbol{X}_i - \overline{\boldsymbol{X}})^T.$$ Consider the following two statements: (I) The distribution of 19 $\boldsymbol{S}$ is $W_6(19, \boldsymbol{\Sigma})$ (Wishart distribution of order 6 with 19 degrees of freedom). (II) The distribution of $(\boldsymbol{X_3} - \boldsymbol{\mu})^T \boldsymbol{S}^{-1} (\boldsymbol{X_3} - \boldsymbol{\mu})$ is χ_6^2 (Chi-square distribution with 6 degrees of freedom). Then which of the above statements is/are true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)

Q.18	Let $X_1, X_2, \dots, X_{18}$ be a random sample from the distribution $$f(x;\ \theta) = \begin{cases} \frac{2x}{\theta} e^{-x^2/\theta}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$ Let $\chi^2_{\alpha,n}$ denote the value of a Chi-square random variable Y with n degrees of freedom such that $P\left(Y > \chi^2_{\alpha,n}\right) = \alpha$. If $x_1, x_2, \dots, x_{18}$ is a realization of this random sample, then, based on the sufficient statistic $\sum_{i=1}^{18} X_i^2$, which one of the following is a 98% confidence interval for θ?
(A)	$\left(\frac{2\sum_{i=1}^{18} x_i^2}{\chi^2_{0.01,36}}, \frac{2\sum_{i=1}^{18} x_i^2}{\chi^2_{0.99,36}}\right)$
(B)	$\left(\frac{2\sum_{i=1}^{18} x_i^2}{\chi^2_{0.01,18}}, \frac{2\sum_{i=1}^{18} x_i^2}{\chi^2_{0.99,18}}\right)$
(C)	$\left(\frac{\sum_{i=1}^{18} x_i^2}{\chi^2_{0.01,36}}, \frac{\sum_{i=1}^{18} x_i^2}{\chi^2_{0.99,36}}\right)$
(D)	$\left(\frac{\sum_{i=1}^{18} x_i^2}{\chi^2_{0.01,18}}, \frac{\sum_{i=1}^{18} x_i^2}{\chi^2_{0.99,18}}\right)$

Q.19	Let $X_1, X_2, \dots, X_n$ be a random sample from a population $f(x;\theta)$, where θ is a parameter. Then which one of the following statements is NOT true?
(A)	$\sum_{i=1}^{n} X_i$ is a complete and sufficient statistic for θ, if $$f(x;\theta) = \frac{e^{-\theta}\theta^x}{x!}\ , \quad x = 0, 1, 2, \dots, \quad \text{and} \quad \theta > 0$$
(B)	$(\sum_{i=1}^{n} X_i\,,\ \sum_{i=1}^{n} X_i^2)$ is a complete and sufficient statistic for θ, if $$f(x;\theta) = \frac{1}{\sqrt{2\pi}\,\theta} e^{-\frac{1}{2\theta^2}(x-\theta)^2}, -\infty < x < \infty, \ \ \theta > 0$$
(C)	$f(x;\theta) = \theta x^{\theta-1}, 0 < x < 1, \ \theta > 0$ has monotone likelihood ratio property in $\prod_{i=1}^{n} X_i$
(D)	$X_{(n)} - X_{(1)}$ is ancillary statistic for θ if $f(x;\theta) = 1, \ 0 < \ \theta < x < \theta + 1$, where $X_{(1)} = \min\{X_1, X_2, \dots, X_n\}$ and $X_{(n)} = \max\{X_1, X_2, \dots, X_n\}$

Q.20	A random sample $X_1, X_2, \ldots, X_6$ of size 6 is taken from a Bernoulli distribution with the parameter θ. The null hypothesis $H_0: \theta = \frac{1}{2}$ is to be tested against the alternative hypothesis $H_1: \theta > \frac{1}{2}$, based on the statistic $Y = \sum_{i=1}^{6} X_i$. If the value of Y corresponding to the observed sample values is 4, then the p-value of the test statistic is
(A)	$\frac{21}{32}$
(B)	$\frac{9}{64}$
(C)	$\frac{11}{32}$
(D)	$\frac{7}{64}$

Q.21	Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of positive real numbers satisfying $$\frac{8}{a_{n+1}} = \frac{7}{a_n} + \frac{a_n^2}{343}, \qquad n \geq 1$$ with $a_1 = 3$ and $a_n < 7$ for all $n \geq 2$. Consider the following statements: (I) $\{a_n\}$ is monotonically increasing. (II) $\{a_n\}$ converges to a value in the interval $[3, 7]$. Then which of the above statements is/are true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)
Q.22	Let $\boldsymbol{M}$ be any square matrix of arbitrary order n such that $\boldsymbol{M}^2 = \boldsymbol{0}$ and the nullity of $\boldsymbol{M}$ is 6. Then the maximum possible value of n **(in integer)** is ______

Q.23	Consider the usual inner product in $\mathbb{R}^4$. Let $\boldsymbol{u} \in \mathbb{R}^4$ be a unit vector orthogonal to the subspace $$S = \{(x_1, x_2, x_3, x_4)^T \in \mathbb{R}^4 \mid x_1 + x_2 + x_3 + x_4 = 0\}.$$ If $\boldsymbol{v} = (1, -2, 1, 1)^T$, and the vectors $\boldsymbol{u}$ and $\boldsymbol{v} - \alpha\boldsymbol{u}$, $\alpha \in \mathbb{R}$, are orthogonal, then the value of α^2 **(rounded off to two decimal places)** is equal to _______
Q.24	Let $\{B(t)\}_{t\geq 0}$ be a standard Brownian motion and let $\Phi(\cdot)$ be the cumulative distribution function of the standard normal distribution. If $$P\left(\left(B(2) + 2B(3)\right) > 1\right) = 1 - \Phi\left(\frac{1}{\sqrt{\alpha}}\right), \qquad \alpha > 0,$$ then the value of α **(in integer)** is equal to _______
Q.25	Let X and Y be two independent exponential random variables with $E(X^2) = \frac{1}{2}$ and $E(Y^2) = \frac{2}{9}$. Then $P(X < 2Y)$ **(rounded off to two decimal places)** is equal to _______
Q.26	Let X be a random variable with the probability mass function $p_X(x) = \left(\frac{3}{4}\right)^{x-1}\left(\frac{1}{4}\right)$, $x = 1, 2, 3, \ldots$. Then the value of $$\sum_{n=0}^{\infty} P(n < X \leq n+3)$$ **(rounded off to two decimal places)** is equal to _______

Q.27	Let $X_i,\ i = 1, 2, \ldots, n$, be $i.i.d.$ random variables from a normal distribution with mean 1 and variance 4. Let $S_n = X_1^2 + X_2^2 + \cdots + X_n^2$. If $Var(S_n)$ denotes the variance of S_n, then the value of $$\lim_{n\to\infty}\left(\frac{Var(S_n)}{n} - \left(\frac{E(S_n)}{n}\right)^2\right)$$ **(in integer)** is equal to ________
Q.28	At a telephone exchange, telephone calls arrive independently at an average rate of 1 call per minute, and the number of telephone calls follows a Poisson distribution. Five time intervals, each of duration 2 minutes, are chosen at random. Let p denote the probability that in each of the five time intervals at most 1 call arrives at the telephone exchange. Then $e^{10}p$ **(in integer)** is equal to ________
Q.29	Let X be a random variable with the probability density function $$f(x) = \begin{cases} c(x - [x]), & 0 < x < 3, \\ 0, & \text{elsewhere}, \end{cases}$$ where c is a constant and $[x]$ denotes the greatest integer less than or equal to x. If $A = \left[\frac{1}{2}, 2\right]$, then $P(X \in A)$ **(rounded off to two decimal places)** is equal to _______

Q.30	Let X and Y be two random variables such that the moment generating function of X is $M(t)$ and the moment generating function of Y is $$H(t) = \left(\frac{3}{4}e^{2t} + \frac{1}{4}\right)M(t),$$ where $t \in (-h, h),\ h > 0$. If the mean and the variance of X are $\frac{1}{2}$ and $\frac{1}{4}$, respectively, then the variance of Y **(in integer)** is equal to ________
Q.31	Let $X_{i,}\ i = 1, 2, \dots n,$ be $i.i.d.$ random variables with the probability density function $$f_X(x) = \begin{cases} \frac{1}{\sqrt{2}\,\Gamma\left(\frac{1}{6}\right)} x^{-5/6}\ e^{-x/8}, & 0 < x < \infty, \\ 0, & \text{elsewhere}, \end{cases}$$ where $\Gamma(\cdot)$ denotes the gamma function. Also, let $\bar{X}_n = \frac{1}{n}(X_1 + X_2 + \cdots + X_n)$. If $\sqrt{n}\left(\bar{X}_n\,(3 - \bar{X}_n) - \frac{20}{9}\right)$ converges to $\mathrm{N}(0, \sigma^2)$ in distribution, then σ^2 **(rounded off to two decimal places)** is equal to ________
Q.32	Consider a Poisson process $\{X(t), t \geq 0\}$. The probability mass function of $X(t)$ is given by $$f(t) = \frac{e^{-4t}\,(4t)^n}{n!}, \qquad n = 0, 1, 2, \dots$$ If $C(t_1, t_2)$ is the covariance function of the Poisson process, then the value of $C(5, 3)$ **(in integer)** is equal to _______

Q.33	A random sample of size 4 is taken from the distribution with the probability density function $$f(x;\theta)=\begin{cases}\dfrac{2(\theta-x)}{\theta^2}, & 0<x<\theta,\\ 0, & \text{elsewhere.}\end{cases}$$ If the observed sample values are $6, 5, 3, 6$, then the method of moments estimate **(in integer)** of the parameter θ, based on these observations, is ______
Q.34	A company sometimes stops payments of quarterly dividends. If the company pays the quarterly dividend, the probability that the next one will be paid is 0.7. If the company stops the quarterly dividend, the probability that the next quarterly dividend will not be paid is 0.5. Then the probability **(rounded off to three decimal places)** that the company will not pay quarterly dividend in the long run is ______
Q.35	Let $X_1, X_2, \dots, X_8$ be a random sample taken from a distribution with the probability density function $$f_X(x)=\begin{cases}\dfrac{x}{8}, & 0<x<4,\\ 0, & \text{elsewhere.}\end{cases}$$ Let $F_8(x)$ be the empirical distribution function of the sample. If α is the variance of $F_8(2)$, then 128α **(in integer)** is equal to ______

Q.36 – Q.65 Carry TWO marks Each

Q.36	Let $\boldsymbol{M}$ be a 3×3 real symmetric matrix with eigenvalues $-1, 1, 2$ and the corresponding unit eigenvectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$, respectively. Let $\boldsymbol{x}$ and $\boldsymbol{y}$ be two vectors in $\mathbb{R}^3$ such that $$\boldsymbol{Mx} = \boldsymbol{u} + 2(\boldsymbol{v} + \boldsymbol{w}) \quad \text{and} \quad \boldsymbol{M}^2\boldsymbol{y} = \boldsymbol{u} - (\boldsymbol{v} + 2\boldsymbol{w}).$$ Considering the usual inner product in $\mathbb{R}^3$, the value of $\lvert\boldsymbol{x} + \boldsymbol{y}\rvert^2$, where $\lvert\boldsymbol{x} + \boldsymbol{y}\rvert$ is the length of the vector $\boldsymbol{x} + \boldsymbol{y}$, is
(A)	1.25
(B)	0.25
(C)	0.75
(D)	1

Q.37	Consider the following infinite series: $$S_1 := \sum_{n=0}^{\infty} (-1)^n \frac{n}{n^2+4} \quad \text{and} \quad S_2 := \sum_{n=0}^{\infty} (-1)^n \left(\sqrt{n^2+1} - n\right).$$ Which of the above series is/are conditionally convergent?
(A)	S_1 only
(B)	S_2 only
(C)	Both S_1 and S_2
(D)	Neither S_1 nor S_2
Q.38	Let $(3,6)^T, (4,4)^T, (5,7)^T$ and $(4,7)^T$ be four independent observations from a bivariate normal distribution with the mean vector $\boldsymbol{\mu}$ and the covariance matrix $\boldsymbol{\Sigma}$. Let $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\Sigma}}$ be the maximum likelihood estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, respectively, based on these observations. Then $\widehat{\boldsymbol{\Sigma}}\widehat{\boldsymbol{\mu}}$ is equal to
(A)	$\begin{pmatrix} 3.5 \\ 10 \end{pmatrix}$
(B)	$\begin{pmatrix} 7.5 \\ 4 \end{pmatrix}$
(C)	$\begin{pmatrix} 4 \\ 13.5 \end{pmatrix}$
(D)	$\begin{pmatrix} 10 \\ 3.5 \end{pmatrix}$

Q.39	Let $\boldsymbol{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ follow $\mathrm{N}_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$ and $\boldsymbol{\Sigma} = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 2 & a \\ 1 & a & 2 \end{bmatrix}$, where $a \in \mathbb{R}$. Suppose that the partial correlation coefficient between X_2 and X_3, keeping X_1 fixed, is $\frac{5}{7}$. Then a is equal to
(A)	1
(B)	$\frac{3}{2}$
(C)	2
(D)	$\frac{1}{2}$

Q.40	If the line $y = \alpha x$, $\alpha \geq \sqrt{2}$, divides the area of the region $$R := \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq \sqrt{y}, 0 \leq y \leq 2\}$$ into two equal parts, then the value of α is equal to
(A)	$\frac{3}{\sqrt{2}}$
(B)	$2\sqrt{2}$
(C)	$\sqrt{2}$
(D)	$\frac{5}{2\sqrt{2}}$

Q.41	Let (X, Y, Z) be a random vector with the joint probability density function $$f_{X,Y,Z}(x,y,z) = \begin{cases} \frac{1}{3}(2x+3y+z), & 0<x<1, 0<y<1, 0<z<1, \\ 0, & \text{elsewhere.} \end{cases}$$ Then which one of the following points is on the regression surface of X on (Y, Z) ?
(A)	$\left(\frac{4}{7}, \frac{1}{3}, \frac{1}{3}\right)$
(B)	$\left(\frac{6}{7}, \frac{2}{3}, \frac{2}{3}\right)$
(C)	$\left(\frac{1}{2}, \frac{1}{3}, \frac{2}{3}\right)$
(D)	$\left(\frac{1}{2}, \frac{2}{3}, \frac{1}{3}\right)$

Q.42	A random sample X of size one is taken from a distribution with the probability density function $$f(x;\,\theta) = \begin{cases} \frac{2x}{\theta^2}, & 0 < x < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$ If $\frac{X}{\theta}$ is used as a pivot for obtaining the confidence interval for θ, then which one of the following is an 80% confidence interval (confidence limits rounded off to three decimal places) for θ based on the observed sample value $x = 10$?
(A)	$(10.541,\ 31.623)$
(B)	$(10.987,\ 31.126)$
(C)	$(11.345,\ 30.524)$
(D)	$(11.267,\ 30.542)$

Q.43	Let $X_1, X_2, \ldots, X_7$ be a random sample from a normal population with mean 0 and variance $\theta > 0$. Let $$K = \frac{X_1^2 + X_2^2}{X_1^2 + X_2^2 + \cdots + X_7^2}.$$ Consider the following statements: (I) The statistics K and $X_1^2 + X_2^2 + \cdots + X_7^2$ are independent. (II) $\frac{7K}{2}$ has an F-distribution with 2 and 7 degrees of freedom. (III) $E(K^2) = \frac{8}{63}$. Then which of the above statements is/are true?
(A)	(I) and (II) only
(B)	(I) and (III) only
(C)	(II) and (III) only
(D)	(I) only

Q.44	Consider the following statements: (I) Let a random variable X have the probability density function $$f_X(x) = \frac{1}{2} e^{-\lvert x \rvert}, \qquad -\infty < x < \infty.$$ Then there exist $i.i.d.$ random variables X_1 and X_2 such that X and $X_1 - X_2$ have the same distribution. (II) Let a random variable Y have the probability density function $$f_Y(y) = \begin{cases} \frac{1}{4}, & -2 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$ Then there exist $i.i.d.$ random variables Y_1 and Y_2 such that Y and $Y_1 - Y_2$ have the same distribution. Then which of the above statements is/are true?
(A)	(I) only
(B)	(II) only
(C)	Both (I) and (II)
(D)	Neither (I) nor (II)

Q.45	Suppose $X_1,\ X_2, \dots, X_n, \dots$ are independent exponential random variables with the mean $\frac{1}{2}$. Let the notation $i.o.$ denote 'infinitely often'. Then which of the following is/are true?
(A)	$P\left(\left\{X_n > \frac{\epsilon}{2}\log_e n\right\}\ i.o.\right) = 1$ for $0 < \epsilon \leq 1$
(B)	$P\left(\left\{X_n < \frac{\epsilon}{2}\log_e n\right\}\ i.o.\right) = 1$ for $0 < \epsilon \leq 1$
(C)	$P\left(\left\{X_n > \frac{\epsilon}{2}\log_e n\right\}\ i.o.\right) = 1$ for $\epsilon > 1$
(D)	$P\left(\left\{X_n < \frac{\epsilon}{2}\log_e n\right\}\ i.o.\right) = 1$ for $\epsilon > 1$

Q.46	Let $\{X_n\}, n \geq 1$, be a sequence of random variables with the probability mass functions $$p_{X_n}(x) = \begin{cases} \frac{n}{n+1}, & x = 0, \\ \frac{1}{n+1}, & x = n, \\ 0, & \text{elsewhere.} \end{cases}$$ Let X be a random variable with $P(X = 0) = 1$. Then which of the following statements is/are true?
(A)	X_n converges to X in distribution
(B)	X_n converges to X in probability
(C)	$E(X_n) \longrightarrow E(X)$
(D)	There exists a subsequence $\{X_{n_k}\}$ of $\{X_n\}$ such that X_{n_k} converges to X almost surely

Q.47	Let $\boldsymbol{M}$ be any 3×3 symmetric matrix with eigenvalues $1, 2$ and 3. Let $\boldsymbol{N}$ be any 3×3 matrix with real eigenvalues such that $\boldsymbol{MN} + \boldsymbol{N}^T\boldsymbol{M} = 3\boldsymbol{I}$, where $\boldsymbol{I}$ is the 3×3 identity matrix. Then which of the following cannot be eigenvalue(s) of the matrix $\boldsymbol{N}$?
(A)	$\frac{1}{4}$
(B)	$\frac{3}{4}$
(C)	$\frac{1}{2}$
(D)	$\frac{7}{4}$
Q.38	Let $\boldsymbol{M}$ be a 3×2 real matrix having a singular value decomposition as $\boldsymbol{M} = \boldsymbol{USV}^T$, where the matrix $\boldsymbol{S} = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T$, $\boldsymbol{U}$ is a 3×3 orthogonal matrix, and $\boldsymbol{V}$ is a 2×2 orthogonal matrix. Then which of the following statements is/are true?
(A)	The rank of the matrix $\boldsymbol{M}$ is 1
(B)	The trace of the matrix $\boldsymbol{M}^T\boldsymbol{M}$ is 4
(C)	The largest singular value of the matrix $(\boldsymbol{M}^T\boldsymbol{M})^{-1}\boldsymbol{M}^T$ is 1
(D)	The nullity of the matrix $\boldsymbol{M}$ is 1

Q.49	Let X be a random variable such that $$P\left(\frac{a}{2\pi}X \in \mathbb{Z}\right) = 1, \quad a > 0,$$ where $\mathbb{Z}$ denotes the set of all integers. If $\phi_X(t), t \in \mathbb{R}$, denotes the characteristic function of X, then which of the following is/are true?
(A)	$\phi_X(a) = 1$
(B)	$\phi_X(\cdot)$ is periodic with period a
(C)	$\|\phi_X(t)\| < 1$ for all $t \neq a$
(D)	$\int_0^{2\pi} e^{-itn}\, \phi_X(t)dt = \pi\, P\left(X = \frac{2\pi n}{a}\right), n \in \mathbb{Z},\ i = \sqrt{-1}$
Q.50	Which of the following real valued functions is/are uniformly continuous on $[0, \infty)$?
(A)	$\sin^2 x$
(B)	$x \sin x$
(C)	$\sin(\sin x)$
(D)	$\sin(x \sin x)$

Q.51 Two independent random samples, each of size 7, from two populations yield the following values:

Population 1	18	20	16	20	17	18	14
Population 2	17	18	14	20	14	13	16

If Mann-Whitney U test is performed at 5% level of significance to test the null hypothesis H_0: Distributions of the populations are same, against the alternative hypothesis H_1: Distributions of the populations are not same, then the value of the test statistic U **(in integer)** for the given data, is ______

Q.52 Consider the multiple regression model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \epsilon,$$

where ϵ is normally distributed with mean 0 and variance $\sigma^2 > 0$, and $\beta_0, \beta_1, \beta_2, \beta_3$ are unknown parameters. Suppose 52 observations of (Y, X_1, X_2, X_3) yield sum of squares due to regression as 18.6 and total sum of squares as 79.23. Then, for testing the null hypothesis $H_0: \beta_1 = \beta_2 = \beta_3 = 0$ against the alternative hypothesis $H_1: \beta_i \neq 0$ for some $i = 1, 2, 3$, the value of the test statistic **(rounded off to three decimal places)**, based on one way analysis of variance, is ________

Q.53	Suppose a random sample of size 3 is taken from a distribution with the probability density function $$f(x) = \begin{cases} 2x, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$ If p is the probability that the largest sample observation is at least twice the smallest sample observation, then the value of p **(rounded off to three decimal places)** is _______
Q.54	Let a linear model $Y = \beta_0 + \beta_1 X + \epsilon$ be fitted to the following data, where ϵ is normally distributed with mean 0 and unknown variance $\sigma^2 > 0$. x_i: 0, 1, 2, 3, 4 y_i: 3, 4, 5, 6, 7 Let $\hat{Y}_0$ denote the ordinary least-square estimator of Y at $X = 6$, and the variance of of $\hat{Y}_0 = c\sigma^2$. Then the value of the real constant c **(rounded off to one decimal place)** is equal to _______
Q.55	Let $0, 1, 1, 2, 0$ be five observations of a random variable X which follows a Poisson distribution with the parameter $\theta > 0$. Let the minimum variance unbiased estimate of $P(X \leq 1)$, based on this data, be α. Then $5^4\alpha$ **(in integer)** is equal to _______

Q.56	While calculating Spearman's rank correlation coefficient, based on n observations $\{(x_i, y_i), i = 1,2, \ldots, n\}$ from a paired data, it is found that x_i are distinct for all $i \geq 2$, $x_1 = x_2$, and $\sum_{i=1}^{n} d_i^2 = 19.5$, where $d_i = \text{rank}(x_i) - \text{rank}(y_i)$. Then the minimum possible value of $n^3 - n$ **(in integer)** is ______
Q.57	In a laboratory experiment, the behavior of cats are studied for a particular food preference between two foods A and B. For an experiment, 70% of the cats that had food A will prefer food A, and 50% of the cats that had food B will prefer food A. The experiment is repeated under identical conditions. If 40% of the cats had food A in the first experiment, then the percentage **(rounded off to one decimal place)** of cats those will prefer food A in the third experiment, is ______
Q.58	A random sample of size 5 is taken from a distribution with the probability density function $$f(x;\theta) = \begin{cases} \dfrac{3x^2}{\theta^3}, & 0 < x < \theta, \\ 0, & \text{elsewhere,} \end{cases}$$ where θ is an unknown parameter. If the observed values of the random sample are $3, 6, 4, 7, 5$, then the maximum likelihood estimate of the $\frac{1}{8}$th quantile of the distribution **(rounded off to one decimal place)** is ______

Q.59	Consider a gamma distribution with the probability density function $$f(x;\beta) = \begin{cases} \frac{1}{24\,\beta^5}\, x^4 e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$ with $\beta > 0$. Then, for $\beta = 2$, the value of the Cramer-Rao lower bound **(rounded off to one decimal place)** for the variance of any unbiased estimator of β^2, based on a random sample of size 8 from this distribution, is ______
Q.60	Let $X_1,\ X_2,\ X_3,\ X_4$ be a random sample of size four from a Bernoulli distribution with the parameter θ, $0 < \theta < 1$. Consider the null hypothesis $H_0\colon \theta = \frac{1}{4}$ against the alternative hypothesis $H_1\colon \theta > \frac{1}{4}$. Suppose H_0 is rejected if and only if $X_1 + X_2 + X_3 + X_4 > 2$. If α is the probability of Type I error for the test and $\gamma(\theta)$ is the power function of the test, then the value of $16\alpha + 7\,\gamma\left(\frac{1}{2}\right)$ **(in integer)** is equal to ______
Q.61	Given that $\Phi(1.645) = 0.95$ and $\Phi(2.33) = 0.99$, where $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal random variable. For a random sample $X_1, X_2, \dots, X_n$ from a normal population $\mathrm{N}(\mu, 2^2)$, where μ is unknown, the null hypothesis $H_0\colon \mu = 10$ is to be tested against the alternative hypothesis $H_1\colon \mu = 12$. Suppose that a test that rejects H_0 if the sample mean $\bar{X}$ is large, is used. Then the smallest value of n **(in integer)** such that Type I error is 0.05 and Type II error is at most 0.01, is ______

Q.62	Let $Y_1 < Y_2 < \cdots < Y_n$ be the order statistics of a random sample of size n from a continuous distribution, which is symmetric about its mean μ. Then the smallest value of n **(in integer)** such that $P(Y_1 < \mu < Y_n) \geq 0.99$, is ______
Q.63	If $P(x, y, z)$ is a point which is nearest to the origin and lies on the intersection of the surfaces $z = xy + 5$ and $x + y + z = 1$. Then the distance **(in integer)** between the origin and the point P is ______
Q.64	Let X and Y be random variables such that X is uniformly distributed over $(0, 4)$, and the conditional distribution of Y given $X = x$ is uniformly distributed over $\left(0, \frac{x^2}{4}\right)$. Then $E(Y^2)$ **(rounded off to three decimal places)** is equal to _____
Q.65	Let $\boldsymbol{X} = (X_1, X_2, X_3)^T$ be a random vector with the distribution $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $$\boldsymbol{\mu} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}.$$ Then $E(X_1 \mid (X_2 = 4, X_3 = 7))$ **(in integer)** is equal to ______